# On the Deformational Characteristics of Porous Polymeric Tubes 

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#### Abstract

SYNOPSIS The deformational behavior of the porous polymeric tube in the operational period has been studied. The tubes are prepared from a polymeric composition consisting of lowdensity polyethylene and vulcanized rubber particles. It is established that during operation the tubes undergo only radial elastic deformation. A threshold pressure is necessary to initiate the radial deformation. At a constant pressure, the radial deformation increases with an increase in the polyethylene content in the tube material. The stress necessary to break the tubes during operation is much lower than the tensile strength.


## INTRODUCTION

In recent years, porous polymeric tubes appeared to be very prospective for use in the field of agriculture. Enter Corporation, a U.S. firm, produces porous polymeric tubes by the trade name "leaky pipe" for regulated irrigation. The irrigation is performed by the water permeating through the tube wall. The rate of permeation is controlled by maintaining the difference between the internal water pressure and the external pressure. The most important properties of the tubes are permeability and mechanical strength. The latter is usually given by the tensile strength at break. But the basic drawback of the porous polymeric tubes is that during the operational process, under the action of internal water pressure, they undergo certain radial deformation that increases with an increase in pressure, and at a critical value of pressure and radial deformation, the tubes break. Thus, the mechanical strength at radial deformation appears to be a basic parameter for the complete characterization of the porous polymeric tubes. An analogous parameter exists in the field of polymeric membranes operating in pressure-driven processes, where different authors propose different

[^0]criteria for the mechanical stability: flux decline rate at a constant pressure, ${ }^{1}$ hysteresis area described by the relationship flux pressure in direct and reverse course, ${ }^{2}$ or the change in membrane thickness in absence ${ }^{3,4}$ and in presence of flux ${ }^{5,6}$ through the membrane. During the pressure-driven process, the membranes become compact, and as a result, the porosity as well as permeability decreases. The membranes undergo elastic, viscoelastic, and plastic deformations. The porosity of the porous polymeric tubes is very low and its changes in the process of operation is negligible. Under the action of internal water pressure, the tubes undergo certain radial deformation and their thickness decreases. Thus, the pressure gradient through the tube wall increases, and as a result, the flux increases. In the present paper, we have made an attempt to determine the nature as well as the value of deformation based on the flux data. On the basis of the calculated values of the radial deformation, the mechanical stability of some porous tubes has been evaluated.

## THEORY

The flux measured as a quantity of fluid passing through the unit external surface of the tube per unit time may be described by the equation of Hagen-Poiseuille ${ }^{7}$ :

$$
\begin{align*}
J & =\frac{\beta \cdot r^{2}}{8 \mu} \cdot \frac{\Delta P}{l}  \tag{1a}\\
\beta & =n \cdot \pi \cdot r^{2} \tag{1b}
\end{align*}
$$

where $J$ is the flux through the tube wall, $\mathrm{m}^{3} \mathrm{~m}^{-2}$ $\mathrm{h}^{-1} ; \beta$, porosity of the tube; $n$, number of pores per unit area, $\mathrm{m}^{-2} ; r$, average radius of the pores, $\mathrm{m} ; \mu$, fluid viscosity, Pas; $\Delta P$, the pressure difference between the internal and external surfaces of the tube, Pa ; and $l$, tube thickness measured as a difference between the external and internal radii, $m$.

Equation (1) shows that for the absolutely undeformable tube the flux would be proportional to the pressure difference between the two surfaces of the tube. But, in practice, the relationship flux pressure for the tube always shows a positive deviation from linearity-an effect attributed to the change in the thickness and porosity of the tube. As the porosity of the tubes is very low (about $12 \%$ ), the change in thickness plays the decisive role for the deviation.

When a relatively long open cylinder is subjected to internal and external pressure, every point in the cylinder experiences certain tangential as well as radial stresses, ${ }^{8,9}$ leading to the corresponding deformations. The different behavior of the porous tubes as compared with the cylinder consists of the fact that the permeating fluid exerts some pressure on the pore wall and thus counteracts the tangential stress. Most probably, they compensate each other and the tangential deformation during operation is practically negligible. The counteraction of the permeating fluid toward the tangential stress is described by Islam and Dimov ${ }^{6}$ in the membrane process. Therefore, during the operational period, the deformation is basically radial.

As it is very difficult to determine the inner diameter of the tube during the operational period, the flux data may be used to calculate the tube thickness. If the tube would not have undergone any radial deformation, the flux $J_{0}$ might be described by the equation of Hagen-Poiseuille as follows:

$$
\begin{equation*}
J_{0}=\frac{\beta_{0} \cdot 0^{2}}{8 \mu} \cdot \frac{\Delta P}{l_{0}} \tag{2}
\end{equation*}
$$

where $\beta_{0}, r_{0}$, and $l_{0}$ are, respectively, the porosity, pore radius, and tube thickness at time $\tau=0$ (i.e., before the application of pressure).

From Eqs. (1) and (2), we obtain

$$
\begin{equation*}
\frac{l}{l_{0}}=\frac{J_{0}}{J} \frac{\beta^{2}}{\beta_{0}^{2}} \tag{3}
\end{equation*}
$$

Considering the negligible change in porosity ( $\beta$ $\approx \beta_{0}$ ), eq. (3) is converted into eq. (4):

$$
\begin{equation*}
\frac{l}{l_{0}} \approx \frac{J_{0}}{J} \tag{4}
\end{equation*}
$$

The radial deformation $\epsilon_{r}$ at a definite pressure difference $\Delta P$ may be determined from the following relationship:

$$
\begin{equation*}
\epsilon_{r}=\frac{l}{l_{0}}-1 \tag{5a}
\end{equation*}
$$

or

$$
\begin{equation*}
\epsilon_{r}=\frac{J_{0}}{J}-1 \tag{5b}
\end{equation*}
$$

Thus, following the flux through the tube wall, the radial deformation may be calculated as a function of pressure.

## EXPERIMENTAL

## Preparation of Porous Tubes

A polymeric composition consisting of low-density polyethylene (melt index $4.1 \mathrm{~g} / 10 \mathrm{~min}$, product of Neftochim, Bulgaria) and vulcanized rubber particles (sizes $0.25-1.00 \mathrm{~mm}$ ) was homogenized in a mechanical mixer. The porous tubes (external diameter, 20 mm ; internal diameter, 15 mm ) were prepared from the polymeric mixture with the help of an extruder. Details about the preparation method are described in Ref. 10.

## Tensile Strength Measurement

The tensile strength and the corresponding deformation at break were measured at room temperature and at a deformation rate of $100 \mathrm{~mm} / \mathrm{min}$. The samples were 100 mm in length with a working portion of 60 mm .

## Flux Measurement

All the tubes were pretreated with an internal water pressure of $0.34 \mathrm{MPa}(\Delta P=0.24 \mathrm{MPa}$ ) for 5 min (unless otherwise stated) to ensure the wetting of the pores. Then, the flux measurement was performed with the tubes 5 m long, at a constant pressure throughout the length of the tube. Registration of the flux was started after the fixation of pressure
for 5 min . The permeate was collected for 10 min . At every internal pressure, the external diameter of the tube was measured and the flux was calculated taking into account the corresponding external surface.

## RESULTS AND DISCUSSION

To verify the nature of the radial deformation (whether elastic, viscoelastic, or plastic), the productivity $Q$ (measured as the quantity of fluid permeated at constant pressure through the unit external surface) is represented as a function of operational period $\tau$ (Fig. 1). As seen from the figure, the productivity increases linearly with time and the corresponding flux $J=d Q / d \tau$ is independent of $\tau$. From the observed relationship it may be concluded that the deformation is mostly elastic. For the development of viscoelastic and plastic deformations (which are developed in time), the tube wall would have become thinner and thinner and the relation$\operatorname{ship} Q=f(\tau)$ would have deviated from linearity (discrete line) and the corresponding flux would have appeared as a function of time. The constancy of the external diameter during operation and the regeneration of the tube thickness immediately after the pressure has been withdrawn also confirm the elastic nature of the radial deformation.

The relationship of flux vs. pressure is represented in Figure 2. The tube was not pretreated with high water pressure and initially it was dry. For the
first cycle, curve 3 (in reverse course) stands over curve 2 (in direct course). This phenomenon may be explained with by fact that the porous polymeric tubes contain pores with a broad pore-size distribution. The pores with different sizes become permeable depending on the pressure. The relationship between the pore radius $r$ and the pressure $P$ necessary to initiate permeation through it is given by the equation of Kantor ${ }^{1}$ :

$$
\begin{equation*}
r=\frac{2 \sigma \cos \theta}{P} \tag{6}
\end{equation*}
$$

where $\sigma$ is the surface tension of water, and $\theta$, the contact angle (water/tube material).

Thus, with an increase in pressure, more and more new pores become permeable. During the reverse course, the small pores are already wetted and the capillary repulsion forces cease to counteract the water flux through them, and as a result, the flux during the reverse course is higher than that in the direct course. When the cycle was repeated, the relationship follows curve 3 in direct as well as in reverse course. The deviation of the flux from linearity with the increase in pressure described by curve 2 is attributed to the gradual wetting of the smaller pores as well as to the change in the tube thickness. Once the pores are wetted, the capillary repulsion forces cease to counteract the flow of water through the tube wall. Thus, the same relationship described by curve 3 is due only to the change in the tube thickness. For that reason, to study the relationship


Figure 1 Productivity $Q$ (1) and the flux $J$ (2) through the tube wall as a function of the operational period $\tau$. Pressure difference, 0.24 MPa . Polyethylene content in the tube material, $20 \%$.


Figure 2 Flux $J$ as a function of the pressure difference $\Delta P$ through the tube wall. Curves 2 and 3 correspond to direct and reverse courses, respectively. Polyethylene content in the tube material, $25 \%$.
flux pressure, the tubes were pretreated for 5 min at a pressure difference of 0.24 MPa through the tube wall and, thus, the influence of capillary forces during experiment was avoided to a great extent. Under such a condition, for all the tubes, the relationship flux vs. pressure will be described only by curve 3. Curve 1 (Fig. 2) was obtained by extrapo-


Figure 3 Radial deformation $\epsilon_{r}$ as a function of pressure $\Delta P$. (3) Polyethylene content in the tube material: (1) $20 \%$, (2) $25 \%$, and (3) $30 \%$.
lation to curve 3 at $\Delta P \rightarrow 0$. In absence of the capillary forces, if the tube was undeformable, the relationship flux pressure would have followed curve 1. To calculate the radial deformation [eq. $5(\mathrm{~b})$ ], the value of $J$ and $J_{0}$ were taken, respectively, from curves 3 and 1.

To establish the relationship between radial deformation and pressure, it is necessary to draw the curves of flux pressure (similar to curves 1 and 3 in Fig. 2 ) independently for different tubes. The radial deformation $\epsilon_{r}$ [calculated according to eq. (5b)] as a function of the pressure difference $\Delta P$ through the tube wall is represented in Figure 3. The mathematical expression of the relationship may be described by the following empirical equation:

$$
\begin{equation*}
-\epsilon_{r}=-a+b \Delta P \quad \text { for } b \Delta P>a \tag{7}
\end{equation*}
$$

where $a$ and $b$ depend on the composition of the tube material.

Figure 3 shows that up to a certain pressure the tubes do not undergo appreciable deformation, and after that, $-\epsilon_{r}$ increases linearly with the pressure. At a constant pressure, $-\epsilon_{r}$ increases with an increase in the polyethylene content in the composition.

The mechanical strengths for elongation and radial deformation are compared in Table I. The flux through the tube wall cannot be determined at the moment of break, and, therefore, the radial defor-

Table I Comparison between the Mechanical Parameters of the Porous Polymeric Tubes for Elongation and Radial Deformation

|  | Mechanical Parameters at Break |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Composition of |  |  | Radial Stress | Radial <br> (he Tube Material: |
| Polyethylene (\%) | Tensile Strength | $\sigma_{e}(\mathrm{MPa})$ |  | $\Delta P(\mathrm{MPa})$ |

mation cannot be calculated by eq. (5b). Knowing the value of pressure at which the tubes break, the corresponding radial deformation was determined either from eq. (7) or by extending the curves in Figure 3 to the values of $\epsilon_{r}$ corresponding to the pressures at break. It is obvious from the table that the values of both types of deformation at break are of the same order; but the stress necessary to break the tubes is six- to ninefold lower for radial deformation than that for elongation.

## CONCLUSION

During the operation, the porous polymeric tubes undergo mainly elastic deformation. Some threshold pressure is necessary to initiate the radial deformation. At a constant pressure, the radial deformation increases with an increase in the polyethylene content in the tube material. The stress necessary to break the tubes during operation is much lower than the tensile strength at break.

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